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Exam Two: MTH 221, Spring 2017

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SCORE = $\frac{3\cancel{7}\,6}{\cancel{39}\,36}$ Excellent

QUESTION 1. (10 points) Let $T : R^3 \to R^4$ be a linear transformation with standard matrix representation $M = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 3 \\ -1 & 1 & -1 \end{bmatrix}$

$R_1+R_2 \to R_2$, $-2R_1+R_3 \to R_3$, $R_1+R_4 \to R_4$

$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \to$ leader col 1 and 3

(i) Find the Range of T

Range of T = Col space of M.

Range (T) = span$\{(1,-1,2,-1), (1,-1,3,-1)\}$

2

(ii) Find the independent-number of Range(T) (i.e., dim(Range(T))

Span of Two indp points, dim Range(T) = 2

2

(iii) Find Z(T) (i.e., Ker(T) or null space of T)

should be.

3 variables dim Ker(T) = 1 Rank = 2

$X_3 = 0$

$X_1 = X_2$

$X_2 \in \mathbb{R}$.

$Z(T) = \{(x_2, x_2, 0) / x_2 \in \mathbb{R}\}$

$Z(T) = \text{Span}\{(1,1,0)\} \to$ one point

correct.

2

(iv) Find $T(0, 3, 0)$

$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 3 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} = 0 \begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 1 \\ -2 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ -1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ -6 \\ 3 \end{bmatrix}$

$T(0,3,0) = (-3, 3, -6, 3)$.

(v) Does the point (2, -2, 5, 4) belong to the Range(T)? Explain

$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ -1 & -1 & -2 \\ 2 & 3 & 5 \\ -1 & -1 & 4 \end{array}\right]$ $R_1+R_2 \to R_2$ $\sim$ $R_1+R_4 \to R_4$ $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 2 & 3 & 5 \\ 0 & 0 & 6 \end{array}\right] \to 0 \neq 6$

No it does not belong. because inconsistant system.

QUESTION 2. (4 points) Let $T : R^3 \longrightarrow R$ be a linear transformation such that $T(1,1,1) = 2$, $T(1,1,0) = 2$, and $T(1,0,1) = 0$. Find $Z(T)$ and find the independent-number of $Z(T)$.[Hint: It is going to be a big mess if you try to find the matrix representation of T first... STARE and think...then you might see an easier way to do it... however it is not wrong if you want to find the matrix representation of T first]

Let $M = [a \; b \; c]$

$a+b+c=2$
$a+c=0$
$a+b=2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \end{array}\right] \xrightarrow[-R_1+R_3 \to R_3]{-R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

$b=2$
$c=0$
$a=0$.

$M = [0 \; 2 \; 0] \longrightarrow Z(T) = Z(M)$ $\boxed{\text{means } b=0 \text{, } a \text{ and } c \in \mathbb{R}.}$

$Z(T) = \text{span}\{(1,0,0),(0,0,1)\}$

$\dim Z(T) = \underline{2}$

QUESTION 3. (4 points) Let $T : R^4 \to R^2$ be a linear transformation such that $T(a_1, a_2, a_3, a_4) = (a_1+3a_4, 3a_1+9a_4)$.

(i) Find the independent-number of Range(T). (i.e., dim(Range(T)))

$$M = \begin{array}{c} \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 3 \\ 3 & 0 & 0 & 9 \end{bmatrix} \end{array} \xrightarrow{-3R_1+R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\text{Span}\{(1,3)\}$ $\quad \underline{\dim \text{Range}(T) = 1}$.

4 variables

(ii) Find the independent-number of Z(T). (i.e., dim(Ker(T)))

$x_1 = -3x_4$

$Z(T) = \{(-3x_4, x_2, x_3, x_4) \mid x_4 \in \mathbb{R}\}$.

x_2 and

3 free variables $\underline{\dim Z(T) = 3}$ + Rank(T) = 1 = 4

QUESTION 4. (4 points) Let $F = span\{(1,4,1,0), (-1,a,0,1), (-1,b,-1,0)\}$ such that the independent-number of F is 3. Find all possible values of a and b. $\to$ Must get 3 leaders.

$$\begin{bmatrix} 1 & 4 & 1 & 0 \\ -1 & a & 0 & 1 \\ -1 & b & -1 & 0 \end{bmatrix} \xrightarrow[R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 4+a & 1 & 1 \\ 0 & 4+b & 0 & 0 \end{bmatrix}$$

To get 3 leaders

$\boxed{\cancel{a = -4} \text{ and } b \neq -4.}$

$a \in \mathbb{R}$

4+a ~~part~~ cant be leader

So need ~~to make~~ 4+b leader.

QUESTION 5. (3 points) Convince me that $D = \{f(x) \in P_4 \mid f(2) = 0 \text{ or } f(-1) = 0\}$ is not a subspace of P_4.

Polynomial in $P_4 = ax^3+bx^2+cx+d$.

$V_1 \in D = x^2 - 4 \to f(2) = 0$

$V_2 \in D = -x + 1 \to f(-1) = 0$

$V_1 + V_2 = x^2 - x - 5$

$[V_1+V_2](2) = 2^2 - 2 - 5$
$= -3 \neq 0 \notin D$.

$[V_1+V_2](-1) = -5 \neq 0$

Axiom 2 fails

D is not a subspace.

QUESTION 6. (5 points) Convince me that $L = \{A \in R^{3\times3} \mid A^T = -A\}$ is a subspace of $R^{3\times3}$. Then find the independent-number of L (i.e., dim(L))

$$A = -A^T = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Main diagonal are all 0 to make $A^T = -A$.

$$L = \text{Span}\left\{ \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \right\} \rightarrow$$

written as a span L is a subspa

$$\text{Fill} \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

$\dim(L) = 3.$

QUESTION 7. (4 points) Given $B = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}, A, C \right\}$ is a basis for $R^{2\times2}$. Find one possibility for A and one possibility for C. SHOW THE WORK

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_1+R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

Hence to form basis for $R^{2\times2}$ need leaders in col 3 and 4

Let $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1+R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow$$ 4 leaders in $\mathbb{R}^4$.

QUESTION 8. (4 points) Let $A = \begin{bmatrix} 2 & a & b & 4 \\ 4 & c & d & 8 \\ 6 & 7 & g & h \end{bmatrix}$ such that A is row-equivalent to $B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Find, a, b, c, d, g, h

(see class, we will talk)

$$\begin{bmatrix} 2 & a & b & 4 \\ 4 & c & d & 8 \\ 6 & 7 & g & h \end{bmatrix} \longrightarrow$$ Col 3 of B all 0 So $\boxed{b = d = g = 0}$

$$\begin{bmatrix} 2 & a & 0 & 4 \\ 4 & c & 0 & 8 \\ 6 & 7 & 0 & h \end{bmatrix} \begin{matrix} -2R_1+R_2 \to R_2 \\ -3R_1+R_3 \to R_3 \end{matrix}$$

$$\begin{bmatrix} 2 & a & 0 & 4 \\ 0 & -2a+c & 0 & 0 \\ 0 & -3a+7 & 0 & -12+h \end{bmatrix} \xrightarrow{\frac{1}{2}R_1,\ R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & a/2 & 0 & 2 \\ 0 & -3a+7 & 0 & -12+h \\ 0 & -2a+c & 0 & 0 \end{bmatrix}$$

$-12 + h = 1$

$h = 13$

$\boxed{c = 2a}$

Quest 8 next pg.

QUESTION 9. (3 points) Convince me that $F = \{A \in R^{3\times4} \mid Rank(A) \le 2\}$ is not a subspace of $R^{3\times4}$

Let B and C be in F where

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$ Rank 1

Rank 2

$$B + C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Rank = 3 $\geq$ 2 $\notin$ F

F $\notin$ Subspace.

Rank(A) = 0 Zero Matrix

Faculty information

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